RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FOURTH SEMESTER EXAMINATION, MAY-JUNE 2013

SECOND YEAR

Date : 27/05/2013 **Mathematics (General)**

Time : 11am – 2pm Paper : IV Full Marks : 75

(Use separate answer books for each group)

Group – A

Answer **any four** questions of the following : [4×5]

1. State comparison test and μ test for the covergence of improper integral. Using μ test show that $\int_0^\infty e^{-x^2}dx$ converges. [2+3]
2. a) If n is a positive integer, prove that $\Gamma(n) = (n-1)!$ [2]
   b) State the relation between Beta function and Gamma function and use it to show that $\int_0^1 x^{3/2}(1-x)^{3/2}dx = \frac{3\pi}{128}$. [3]
3. Evaluate $\iint xy(x+y)\,dx\,dy$ over the area bounded by $y = x^2$ and $y = x$. [5]
4. Find the volume when the curve $r = a(1-\cos\theta)$ is rotated about its initial line. [5]
5. Find the area common to the circle $x^2 + y^2 = 25$ and the parabola $3x^2 = 16y$. [5]
6. Find the surface area generated by the revolution of $y = x^2$ about y –axis between x = 0 and $x = \sqrt{2}$. [5]

Group – B

Answer **any two** questions of the following : [2×5]

7. Solve : $\frac{d^2y}{dx^2} + a^2y = \sec ax$. [5]
8. Solve : $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$. [5]
9. Find the orthogonal trajectories of the family of hypocycloids $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, where a is a variable parameter. [5]

Group – C

Answer **any four** questions from question number 10-15 and **any five** questions from question number 16-23. [9×5]

10. Define independence of two events. If two events A and B are independent, show that their complementary events $\bar{A}$ and $\bar{B}$ are also independent. [1+4]
11. A man takes a step forward with probability 0.7 and backward with probability 0.3. Find the probability that at the end of 13 steps he is 3 step forward from the starting point. [5]
12. If A and B are two events and are not disjoint then prove that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. [5]
13. State Baye's theorem.
    A and B are two weak students of statistics and their probabilities of solving a problem in statistics correctly are $\frac{1}{6}$ and $\frac{1}{8}$ respectively. If the Probability of getting an identical wrong answer from both

A and B is $\frac{1}{525}$ (i.e. both the answers of A and B are same but wrong), then find the probability of getting an identical correct answer from both A and B. (i.c. both the answers of A and B are same and correct). [5]

14. Let X be a continuous random variable with probability density function,

$$f(x)=\begin{cases} ax, 0 \le x \le 1 \\ a, 1 < x \le 2 \\ -ax+3a, 2 < x \le 3 \\ 0, \quad \text{else where} \end{cases}$$

i) Determine the constant a.

ii) Compute $P(X \le 2.5)$. [2+3]

15. Define mean of a distribution. Find the mean of the poisson (μ) distribution. [1+4]

16. Define primary data and secondary data. Draw a suitable pie diagram for the following data expenditures of Government of India for a particular year. [2+3]

Expenditure	Rs. (in millions)
Education	272.6
Health	2774.5
National Defence	3784.6
Rural Developement	1682.2
Others	1768.6

17. The following table exhibits the daily expenditure distribution of 100 Families.

Expenditure (in Rs.) :	0-10	10-20	20-30	30-40	40-50
Number of Families :	14	x	27	y	15

If the mode of the distribution be 24, find x and y. [5]

18. What is the median of a distribution? Prove that mean deviation is least when taken about median. [1+4]

19. Find the mean and standered deviation or age for the following distribution. [5]

Age (in years)	No. of members
20-30	3
30-40	61
40-50	132
50-60	78
60-70	18

20. Given Cov (x, y) = $\frac{4}{5}$, $S_x = 1$, $S_y = \frac{6}{5}$ Compute $r_{xy.}$

For two variable x and y, the equations of regresion of Y on x and that of X on Y are respectively $3x + 2y = 25$ & $6x + y - 30 = 0$.

Find the correlation coefficient. If var(x) = 9, find standard deviation of y. [2+3]

21. What is null hypothesis?

State clearly Type – I and Type – II error. [1+4]

22. Obtain the trend values for the given data by fitting a suitable quadratic trend. [5]

Year	Values(yt)
1983	6096
1984	6379
1985	6774
1986	7327
1987	7516
1988	7863
1989	8427

23. For the following data obtain, i) Laspeyrie's price index number ii) Paasche's price index number & iii) Fisher's price index number for 2003 with 1998 as the base year. [5]

COMMODITY	1998		2003	
	Quantity	Price	Quantity	Price
A	5	2.00	7	4.00
B	7	3.00	10	3.20
C	6	8.00	6	4.50
D	2	1.50	9	2.00
E	4	5.00	5	5.75